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Multicriterion Evolutionary Structural Optimization Using the Weighting and the Global Criterion Methods

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The design of structures using evolutionary structural optimization (ESO) has previously been based on a single criterion. Multiple criteria are incorporated into the ESO process. This is done using a weighting method and a global criterion method. In the present work, two criteria are optimized simultaneously, namely, the maximization of the first mode of natural frequency and the minimization of the mean compliance of the structure (which is inversely proportional to stiffness). Two examples are provided that show that the solutions produced by the weighting method are Pareto optimum. The solutions of the global criterion method also form part of the Pareto optimum. It appears to have a notable similarity to a solution of 50% stiffness: 50% frequency ratio of criteria weighting. The results point toward a means of producing Pareto optimum solutions where there are more than two criteria to be simultaneously optimized.

I. Introduction

UNTIL recently, evolutionary structural optimization (ESO) has been implemented for single or multiple load case structural problems and for many individual optimization criteria such as stress,[1] strain,[1] stiffness,[2] natural frequency,[3] buckling,[4] stress minimization,[5] and heat transfer.[6]

Multiple criterion ESO is a continuation of ESO, whereby different design criteria are combined to drive the optimization process. This may be done from two different criteria to any specified number of criteria. The two objectives that are pursued in this paper are the maximization of stiffness and the maximization of the first mode natural frequency for a general two-dimensional topology optimization. Much pioneering work has been done in the field of topology optimization by Bendsøe and Kikuchi[7] and Bendsøe.[8]

Multicriteria (multiobjective) optimization is the method of forming a solution that satisfies a number of conflicting objectives in the best way possible. That is, multicriteria optimization presents information about the optimum performance of the structure taking into account different criteria.[9,10] Koski[11] defines it in terms of the problem

$$\min_{x \in \Omega}[f_1(\boldsymbol{x}) f_2(\boldsymbol{x}), \ldots, f_i, \ldots, f_m(\boldsymbol{x})]^T \tag{1}$$

where f_i are the different criteria for $i = 1, 2, \ldots, m$; $\boldsymbol{x} = [x_1\ x_2, \ldots, x_n]^T$ represents the vector of design variables; and Ω is the feasible set in the design space R^n.

The solution to the multicriteria problem is known as a Pareto optimum (or a noninferior solution).[10,12] It is not one Pareto solution that exists for a multicriteria problem, but a series of such solutions that make up the optimum.[13] Once this optimum has been reached, any further improvement in one criterion requires a clear tradeoff with at least one other criterion.[14] Koski defines the Pareto optimum as follows: A vector $\boldsymbol{x}^* \in \Omega$ is Pareto optimal for problem (1) if and only if there exists no $\boldsymbol{x} \in \Omega$ such that $f_i(\boldsymbol{x}) \leq f_i(\boldsymbol{x}^*)$ for $i = 1, 2, \ldots, m$ with $f_j(\boldsymbol{x}) < f_j(\boldsymbol{x}^*)$ for at least one j.

In other words, he defined the Pareto optimum as follows: For a pareto optimum, $\boldsymbol{x}^*$ is a Pareto optimal solution if there exists no feasible solution $\boldsymbol{x}$ that can decrease some objective functions without causing at least one objective function increase.[11]

This paper examines the multicriteria optimization problem incorporated into ESO using the weighting method technique and the global criterion method. ESO is a simple concept of optimization that was developed by Xie and Steven[1] in 1992. It works by deleting elements iteratively from a structure modeled by finite elements (FE). The removal of these elements is based on their contributions to the objective of the particular combined criteria, be it to maximize the stiffness throughout the FE structure and/or to maximize the first mode natural frequency.

II. Determination of Sensitivity Numbers for Element Removal

The procedure for the weighting method and the global criterion method of multicriteria optimization begins with the evaluation of the sensitivity numbers for the two criteria described in this section. This is followed by options for amalgamating these two criteria (Sec. III), which shall later be adopted by the evolutionary algorithm for element removal that is presented in Sec. IV.

A. Stiffness Constraint

In the FE method, the static behavior of a structure is given by

$$[\boldsymbol{K}]\{\boldsymbol{d}\} = \{\boldsymbol{P}\} \tag{2}$$

where $[\boldsymbol{K}]$ is the global stiffness matrix and $\{\boldsymbol{d}\}$ and $\{\boldsymbol{P}\}$ are the global nodal displacement and nodal load vectors, respectively. The inverse measure of the overall stiffness of a structure is known as the mean compliance C and is defined as

$$C = \tfrac{1}{2}\{\boldsymbol{P}\}^T\{\boldsymbol{d}\} \tag{3}$$

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Therefore, the single criterion objective to minimize the mean compliance C is mathematically given as

$$\text{minimize}(C) \tag{4}$$

Minimizing the mean compliance maximizes the overall stiffness of a structure. Hence, the stiffness constraint is given in the form $C \leq C_{\text{all}}$, where C_{all} is the prescribed limit for C (Refs. 1 and 15). From this comes the stiffness sensitivity number for problems with an overall stiffness constraint[1]:

$$\alpha_i = \tfrac{1}{2}\{\boldsymbol{u}^i\}^T[\boldsymbol{K}^i]\{\boldsymbol{u}^i\} \tag{5}$$

where $\{\boldsymbol{u}^i\}$ is the displacement vector of the ith element and $[\boldsymbol{K}^i]$ is the stiffness matrix of the ith element. The quantity α_i indicates the change in the strain energy as a result of removing the ith element. To achieve the optimization objective through element removal, the element that has the lowest value of α_i is removed so that the increase in C is minimal.

Note that, although C inevitably goes up with element removal, the optimization process is such that the product of the compliance and the structures' volume V goes down. In other words, the specific stiffness (structures' stiffness K divided by the structures' volume V) is increased optimally.

B. Frequency Constraint

The dynamic behavior of the structure is represented by the following general eigenvalue problem:

$$\big([\boldsymbol{K}] - \omega_n^2[\boldsymbol{M}]\big)\{\boldsymbol{u}_n\} = \{0\} \tag{6}$$

$[\boldsymbol{M}]$ is the global mass matrix, ω_n is the nth natural frequency, and $\{\boldsymbol{u}_n\}$ is the eigenvector corresponding to ω_n. The mathematical representation for the single criterion maximization of the fundamental frequency may, therefore, be given as

$$\text{maximize}(\omega_n) \tag{7}$$

The frequency sensitivity number α_n^i for the nth mode, which is an indicator of the frequency change (or, to be more precise, the change of the square of the frequency) due to the removal of the ith element, is given by Eq. (8) (Ref. 16):

$$\alpha_n^i = (1/m_n)\big\{\boldsymbol{u}_n^i\big\}^T\big(\omega_n^2[\boldsymbol{M}^i] - [\boldsymbol{K}^i]\big)\big\{\boldsymbol{u}_n^i\big\} \tag{8}$$

in which m_n is the modal mass, $\boldsymbol{u}_n^i$ is the element eigenvector, and $[\boldsymbol{K}^i]$ and $[\boldsymbol{M}^i]$ are the stiffness and mass matrices of the removed element. A maximum increase in the chosen frequency occurs as a result of removing elements whose frequency sensitivity number is the highest. A maximum decrease in the chosen frequency occurs as a result of removing elements whose frequency sensitivity number is the lowest. Removal of those elements whose α is close to zero results in very little change, that is, the frequency remains almost unchanged.[16] Note that due to Eq. (6), the sum of α_i over all of the elements must be zero.

For multicriterion ESO, using stiffness sensitivity numbers in collaboration with the frequency sensitivity numbers necessitates a linear scaling in the frequency sensitivity numbers. For the case of the weighting and the global criterion method of multicriterion ESO, this is done to alleviate the problem of combining the stiffness sensitivity number (with element removal based on the lowest sensitivity values) and the frequency sensitivity number (with element removal based on the highest sensitivity values). The shift allows the criteria to be combined into a single weighted criterion [$F_{\text{multicrit}}^i$; Eq. (10)] or a balanced global criterion [$G_{\text{multicrit}}^i$; Eq. (12)] with the removal of elements based on their lowest sensitivity values. The frequency sensitivity number linear shift is attained using the formula

$$\big\{\alpha_n^i\big\}^{\text{new}} = -\Big(\big\{\alpha_n^i\big\}^{\text{old}}\Big) + \big\{\alpha_n^*\big\}^{\text{old}} \tag{9}$$

where $\{\alpha_n^i\}^{\text{new}}$ is the new frequency sensitivity number for the ith element and the nth natural frequency, $\{\alpha_n^i\}^{\text{old}}$ is the old frequency sensitivity number for the ith element and the nth natural frequency,

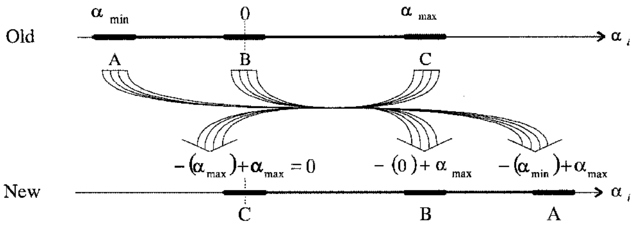

Fig. 1 Change in natural frequency sensitivity numbers.

and $\{\alpha_n^*\}^{\text{old}}$ is the old maximum frequency sensitivity number for the nth natural frequency. Figure 1 illustrates the linear shift in the natural frequency sensitivity number.

III. Optimization Techniques

A. Weighting Method Formulation

The weighting method is used because of its simplicity to produce the whole set of Pareto optima for problem (1) by varying the weighting on each criterion. This is true only for the case where problem (1) is convex.[17] Such a convex case exists for both examples given in Sec. V. The weights given in the weighting method indicate the relative importance of the criteria. When the sensitivity numbers of the two criteria (Sec. II) are obtained, each of these numbers is normalized and is then assigned a weighting factor. These sensitivity numbers are finally added together to form a new single criterion:

$$F_{\text{multicrit}}^i = w_1 R_1^i + w_2 R_2^i + \cdots + w_N R_N^i = \sum_{j=1}^{N} w_j R_j^i \tag{10}$$

where $F_{\text{multicrit}}^i$ is the multiple criteria objective function that determines element removal for each element i; w_j is the jth criteria weighting factor with $w_j \geq 0$ and $j = 1, \ldots, N$; $R_j^i = \alpha_j^i/\alpha_j^*$ is the ratio of the jth criteria sensitivity number α_j^i for each element i to the maximum value of the jth criteria sensitivity number (α_j^*); and N is the total number of criteria.

The criteria weighting is subject to the normalization

$$\sum_{j=1}^{N} w_j = 1 \tag{11}$$

B. Global Criterion Method Formulation

The global criterion[14,18] method, unlike the weighting method, does not allow the designer to select a proportion of weighting allocated to each criterion. Rather, it implicitly allocates equal weighting to all criteria. It is based on the formulation of a metric function that represents the distance between the ideal solution (the minimum value of each criterion) and the optimum solution. The metric function is calculated by

$$G_{\text{multicrit}}^i = \Big[\big(R_1^i - S_1^i\big)^p + \big(R_2^i - S_2^i\big)^p + \cdots + \big(R_N^i - S_N^i\big)^p\Big]^{\frac{1}{p}}$$

$$= \left[\sum_{j=1}^{N}\big(R_j^i - S_j^i\big)^p\right]^{\frac{1}{p}} \tag{12}$$

where $G_{\text{multicrit}}^i$ is the metric multiple criterion objective function that determines element removal for each element i, $S_j^i = \alpha_j^{\min}/\alpha_j^*$ is the ratio of the minimum value of the jth criterion sensitivity number $\alpha_j^{\min}$ to the maximum value of the jth criterion sensitivity number α_j^*, and p is a constant constrained by the condition $1 \leq p \leq \infty$. Typically $p = 1$ or 2. In this study, it was elected to be $p = N = 2$.

IV. Evolutionary Procedure

A. Weighting Method Multicriterion ESO

The ESO method follows a concept that has proved to be very simple and robust. The evolutionary procedure for weighted multicriterion optimization based on ESO for stiffness and natural frequency is given as follows:

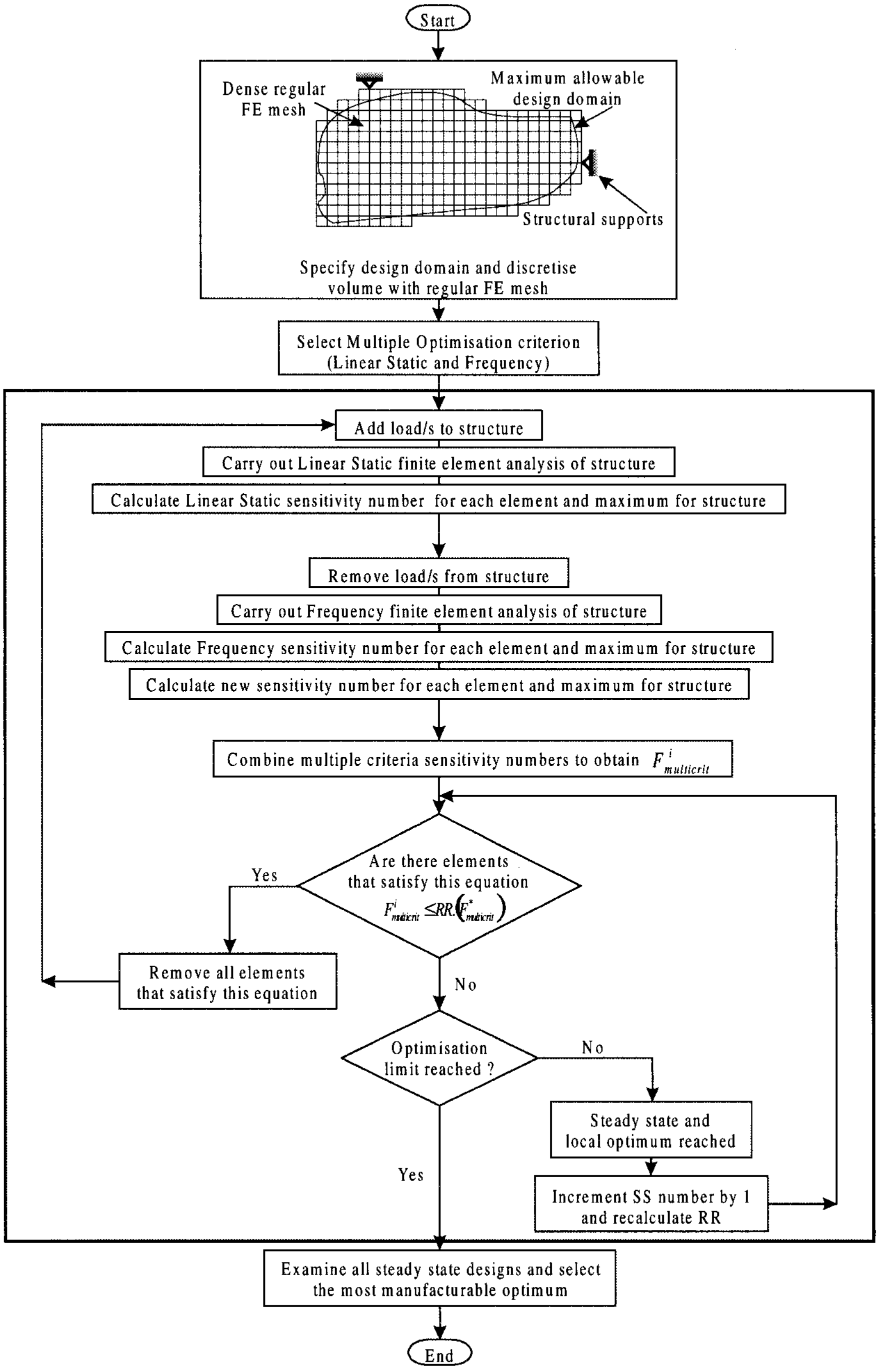

Fig. 2 Flowchart depicting logical steps of weighting method multicriterion ESO.

1) Discretize the structure using a fine mesh of finite elements.
2) Add load(s) to structure.
3) Solve the linear static problem, Eq. (2).
4) Calculate the linear static sensitivity number α_i using Eq. (5).
5) Remove load(s) from structure.
6) Solve the eigenvalue problem, Eq. (6).
7) Calculate the natural frequency sensitivity number α_n^i using Eq. (8).
8) Calculate the new natural frequency sensitivity number $\{\alpha_n^i\}^{\text{new}}$ for linear scaling using Eq. (9).
9) Combine the two criterion sensitivity numbers using Eq. (10) to obtain $F^i_{\text{multicrit}}$.
10) Remove a number of elements that have the lowest values of $F^i_{\text{multicrit}}$ using the inequality

$$F^i_{\text{multicrit}} \leq \text{RR}\left(F^*_{\text{multicrit}}\right) \tag{13}$$

where RR is the current rejection ratio. It is an iterative counter used to dampen or delay the element removal process. It is increased by a defined proportion, typically 0.001, until elements are removed, and is confined to the condition $(0.0 \leq \text{RR} \leq 1.0)$. $F^*_{\text{multicrit}}$ is the maximum $F^i_{\text{multicrit}}$ value that exists in the structure.
11) Repeat steps 2–10 until an optimum is reached.

Figure 2 illustrates the set of logical steps just mentioned.

B. Global Criterion Method Multicriterion ESO

The solution procedure for achieving multicriterion ESO using the global criterion method is similar to that in Sec. IV.A. The only difference is in steps 9 and 10, which should be replaced as follows:
9) Combine the two criterion sensitivity numbers using Eq. (12) to obtain $G^i_{\text{multicrit}}$.
10) Remove a number of elements that have the lowest values of $G^i_{\text{multicrit}}$ using the inequality

$$G^i_{\text{multicrit}} \leq \text{RR}\left(G^*_{\text{multicrit}}\right) \tag{14}$$

where $G^*_{\text{multicrit}}$ is the maximum $G^i_{\text{multicrit}}$ value that exists in the structure.

V. Example Problems

The examples presented in this section are all two-dimensional plane stress problems, with only in-plane vibration considered. The elements used here are of the four-noded linear quadrilateral type. The two driving criteria are the minimization of the mean compliance and the maximization of the first mode of natural frequency.

A. Example 1: Rectangular Plate with Fixed Supports

A rectangular aluminum plate of dimension 0.15×0.1 m is fixed at two diagonal corners, with two horizontal loads (each 100 N) applied on the other two diagonal corners as seen in Fig. 3. These are included for the linear static stress analysis, but are removed for the frequency analysis.

This example is based on a rectangular plate model that was used by Xie and Steven in their investigation of ESO for dynamic problems.[16] The physical data are as follows: Young's modulus $E = 70$ GPa, Poisson's ratio $\nu = 0.3$, thickness $t = 0.01$ m, and density $\rho = 2700$ kg/m^3. The domain is divided into 45×30 square elements.

Figure 4 shows the comparison between the first mode natural frequency and the mean compliance of the structure for a range of different weightings of the criteria and for a 30% volume reduction. Points marked by 5a–5h and 6 correspond to the topologies (and criteria weightings) shown in Figs. 5 and 6, respectively.

For the case where the structure has been designed completely for stiffness ($w_{\text{stiff}} : w_{\text{freq}} = 1.0 : 0.0$), the mean compliance is at the lowest possible value of 0.1837 Nm (see point 5a in Fig. 4), verifying that this configuration is the stiffest possible structure. For 30% of the material removed from the initial structure, the natural frequency for this topology (2666.6 Hz) is the lowest possible frequency.

The mean compliance of the fully stiff optimized design has increased by 4.9% from the mean compliance of the initial design domain (0.1751 Nm). In other words, the stiffness of the structure has decreased due to the removal of 30% of the material from the initial design domain. However, this volume reduction is greater than the reduction in stiffness, meaning that the specific stiffness of the optimized topology is greater than the initial design domain. This is indicated by a decrease in the C multiplied by the volume term ($C \times V$) for the initial structure of from 3.502×10^{-4} Nm^4 [0.1751 Nm $\times$ (100% of 0.002 m^3)] to 2.572×10^{-4} Nm^4 [0.1837 Nm $\times$ (70% of 0.002 m^3)] for the fully stiff topology. Increased specific stiffness is one of the integrated objectives that occur for stiffness optimization as the design evolves.

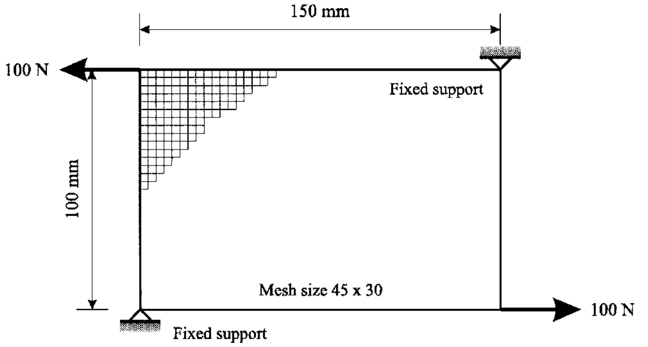

Fig. 3 Initial design domain of rectangular plate under loading with fixed supports; first mode natural frequency, 2498.9 Hz and mean compliance, 0.1751 Nm.

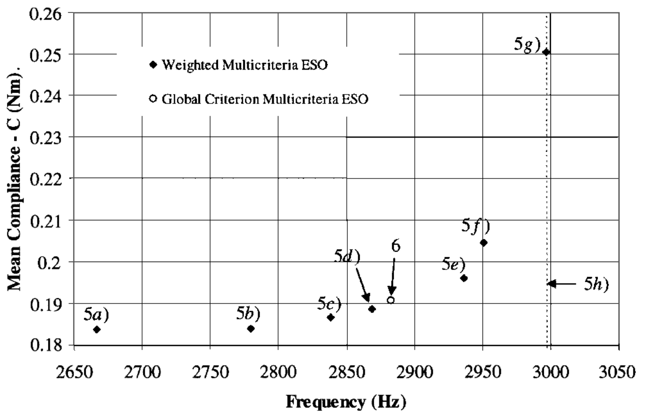

Fig. 4 Plot of first mode natural frequency vs mean compliance with 30% of material removed, where a) = $w_{\text{stiff}} : w_{\text{freq}} = 1.0 : 0.0$, b) = $w_{\text{stiff}} : w_{\text{freq}} = 0.9 : 0.1$, c) = $w_{\text{stiff}} : w_{\text{freq}} = 0.7 : 0.3$, d) = $w_{\text{stiff}} : w_{\text{freq}} = 0.5 : 0.5$, e) = $w_{\text{stiff}} : w_{\text{freq}} = 0.3 : 0.7$, f) = $w_{\text{stiff}} : w_{\text{freq}} = 0.2 : 0.8$, g) = $w_{\text{stiff}} : w_{\text{freq}} = 0.1 : 0.9$, and h) = $w_{\text{stiff}} : w_{\text{freq}} = 0.0 : 1.0$.

When the criteria weightings are changed (i.e., when the frequency weighting is increased and the stiffness weighting is decreased), it can be observed that the natural frequency of the structure increases and that the mean compliance increases, meaning the overall stiffness of the structure decreases. Any improvement of one criterion requires a clear tradeoff with the other. These solutions are all optimal for their own criteria weight allocation, verifying that they all form the Pareto solution. This trend is exponential.

Any deviation of points from this trend is attributed to the evolution history of the criteria weighting not having the topology that exactly matches the specified volume fraction. For example, an evolution history of a specified percentage of stiffness criteria may not exactly have a topology with 30% of the material removed but rather 28 or 31%. The closest volume fraction to the specified one is selected, that is, 31%.

The dotted line on the right-hand side of Fig. 4 represents the frequency that has resulted for the full frequency optimized design ($w_{\text{stiff}} : w_{\text{freq}} = 0.0 : 1.0$). No compliance value can be calculated for this topology (30% of the material removed) because the elements that attach the applied loads to the structure have been removed (see Fig. 5h). Here, the frequency is 2997.1 Hz, a 12.4% increase from the fully stiff design. The natural frequency of the initial design domain is 2498.9 Hz. For 30% of material removed, the topology based on 100% frequency optimization has a higher frequency by 19.9%.

The configuration of the topology driven by the global criterion method is presented in Fig. 6. Its mean compliance is 0.1907 Nm, and its natural frequency is 2882.6 Hz. These values can been observed in Fig. 4, represented by point 6. It has a topology very much like the 50% stiffness: 50% frequency weighted configuration.

B. Example 2: Rectangular Plate with Roller Supports

A structure is to be designed to support nine point loads, each of 200 N, distributed at 0.01-m intervals, under the given boundary conditions shown in Fig. 7. The dimensions for the design domain are 0.8 m in width, 0.5 m in height, and 0.01 m in thickness. Because of symmetry, only half of the design domain is analyzed and is discretized into 40×50 four-noded square elements. The material properties of the elements are Young's modulus $E = 200$ GPa, Poisson's ratio $\nu = 0.3$, and density $\rho = 7000$ kg/m^3. In the analysis, two-dimensional plane stress conditions are assumed.

The Pareto curves of the first mode natural frequencies and the $C \times V$ terms are presented in Fig. 8. Points marked by 9a–9g and 10 correspond to the topologies (and criteria weightings) shown in Figs. 9 and 10, respectively. The product $C \times V$ is used in this example (in contrast to the compliance C being analyzed in the first example) to enable a relative comparison to be made between the different volumes of material removed during the evolutionary procedure. These curves are shown for 10, 20, 30, 35, and 40% of material removed using weighted multicriteria ESO. For each of these volume fractions, the variation in the criteria weighting is illustrated, ranging from 100% stiffness weighting (0% frequency weighting) at point A to 0% stiffness weighting (100% frequency weighting) at point B. The other five points correspond to 10, 30, 50, 70, and 90% in stiffness weighting.

This, however, is not the case for the Pareto optimums with 35 and 40% of the material removed. The 35% case does not include the criteria weighting of 0% stiffness weighting (100% frequency), meaning that the 90% stiffness weighting is denoted by point B. The 40% case excludes the criteria weightings of 10 and 0% stiffness weighting (90 and 100% frequency, respectively), meaning that the 70% stiffness weighting is denoted by point B.

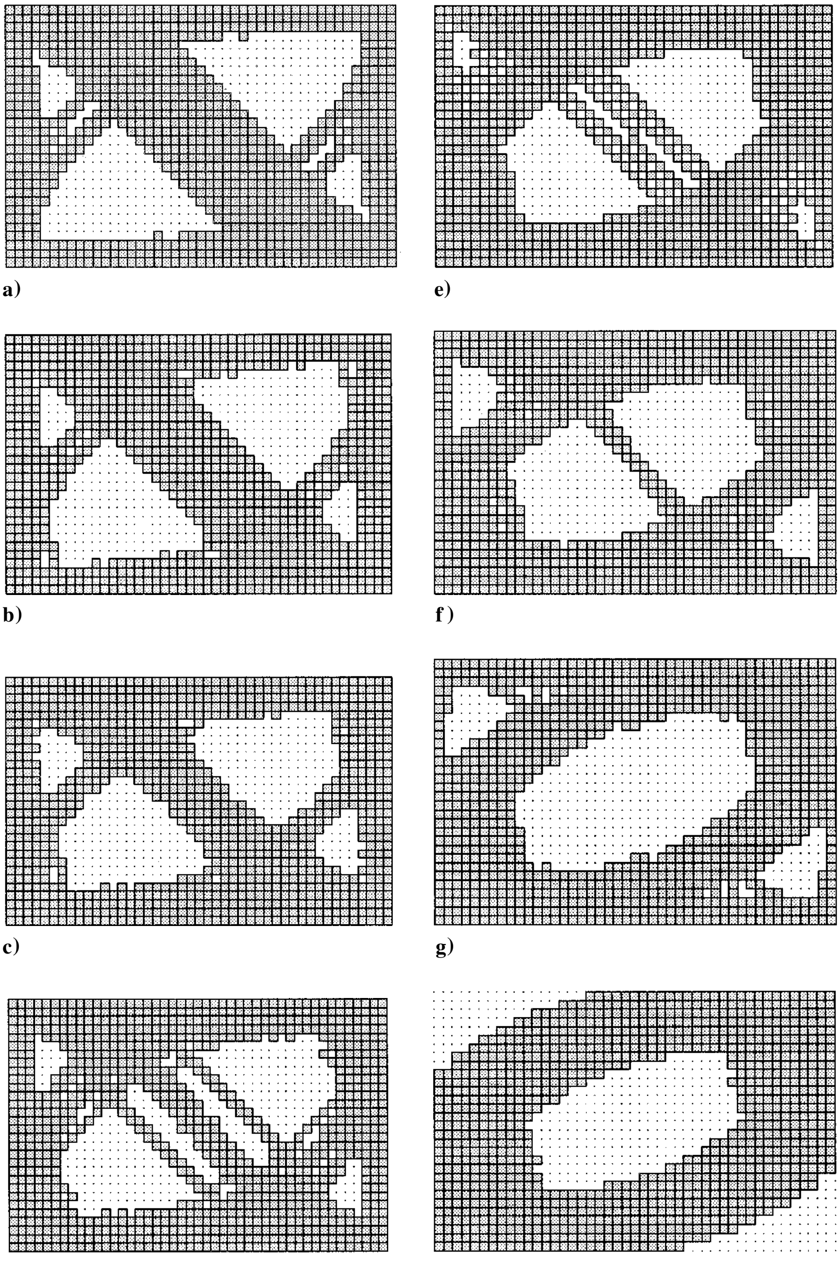

Fig. 5 Optimal designs of rectangular plate for different weighting criteria of stiffness and natural frequency; material removed, 30%.

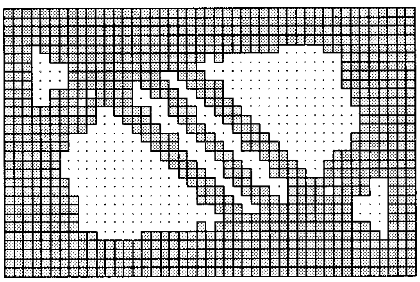

Fig. 6 Optimum design of rectangular plate for global criterion method of multicriterion ESO of stiffness and natural frequency; material removed, 30%.

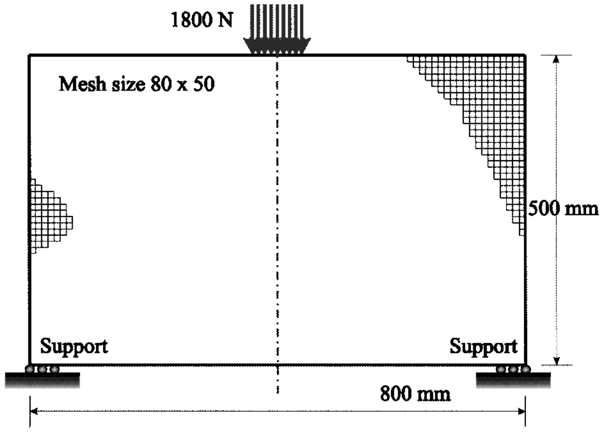

Fig. 7 Initial design domain of rectangular plate under loading with roller supports.

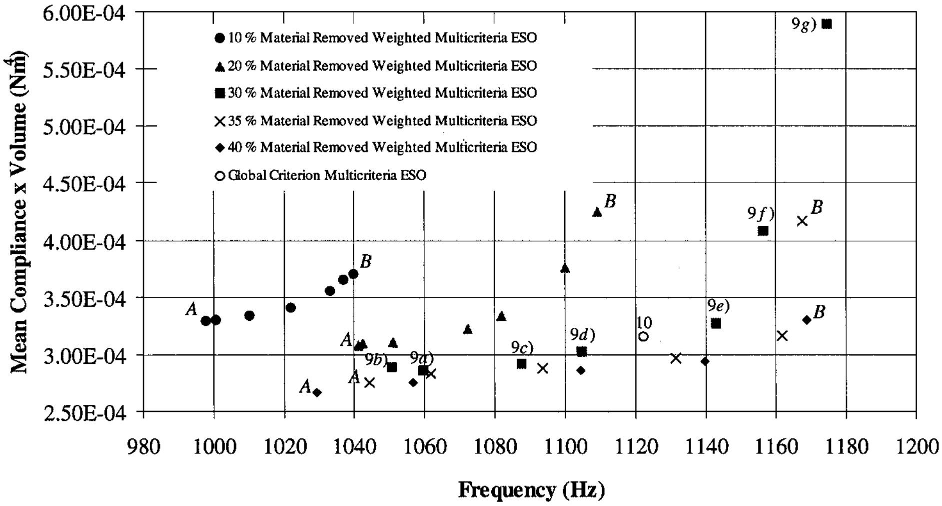

Fig. 8 Plot of first mode natural frequency vs mean compliance times volume ($C \times V$) for varying range in percentage of material removed, where a) = $w_{\text{stiff}} : w_{\text{freq}} = 1.0 : 0.0$, b) = $w_{\text{stiff}} : w_{\text{freq}} = 0.9 : 0.1$, c) = $w_{\text{stiff}} : w_{\text{freq}} = 0.7 : 0.3$, d) = $w_{\text{stiff}} : w_{\text{freq}} = 0.5 : 0.5$, e) = $w_{\text{stiff}} : w_{\text{freq}} = 0.3 : 0.7$, f) = $w_{\text{stiff}} : w_{\text{freq}} = 0.1 : 0.9$, and g) = $w_{\text{stiff}} : w_{\text{freq}} = 0.0 : 1.0$.

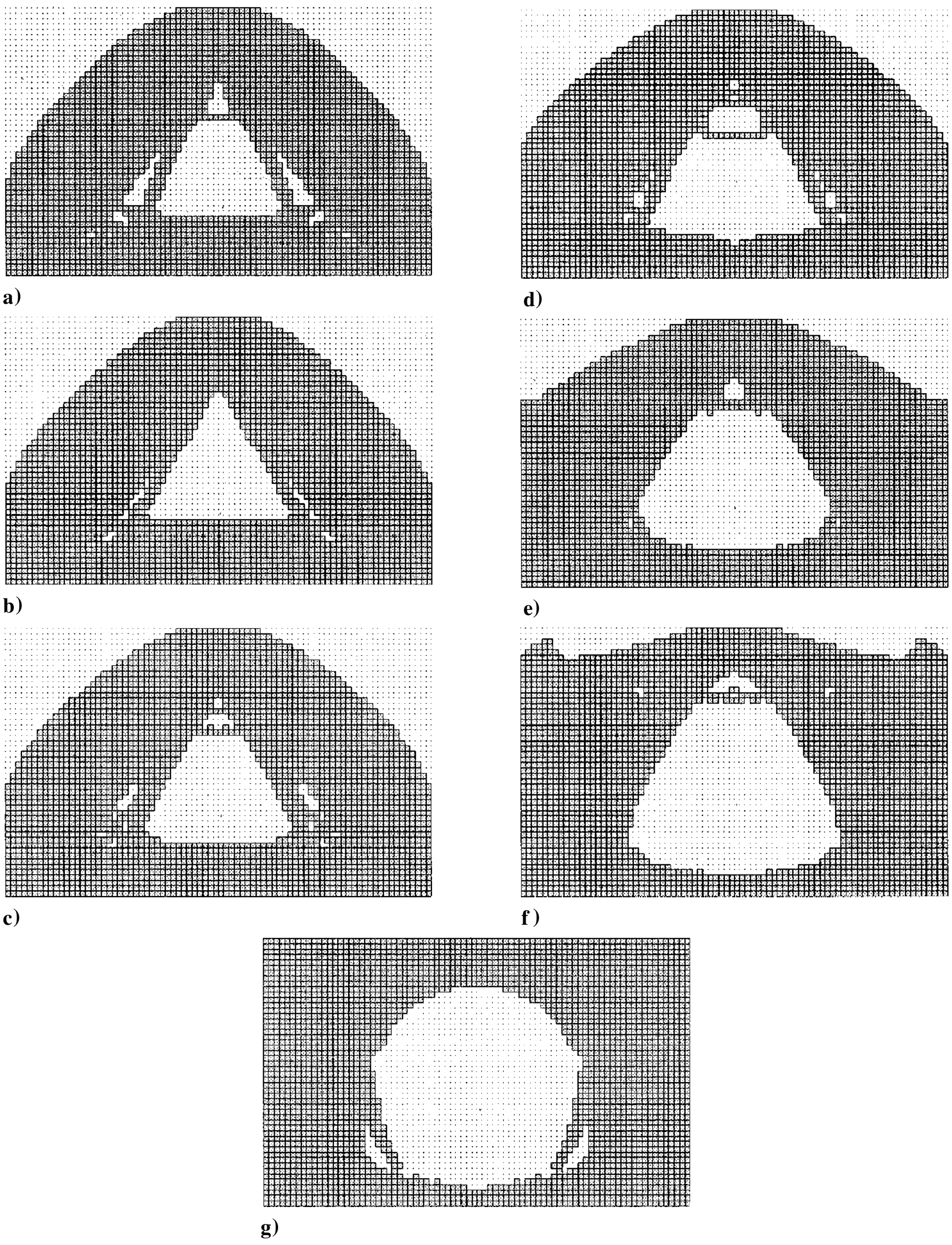

Fig. 9 Optimal designs of roller supported rectangular plate for different weighting criteria of stiffness and natural frequency; material removed, 30%.

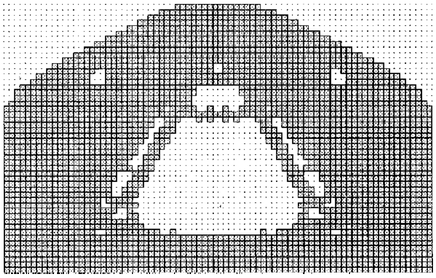

Fig. 10 Optimum design of roller supported rectangular plate for global criterion method of multicriterion ESO of stiffness and natural frequency; material removed, 30%.

The reason that such criterion weightings do not exist for the coinciding volume fractions is that elements were removed in the ESO process until the structure became unstable. In other words, elements that carried the applied loads were removed. When this occurred, the first mode of natural frequency of the design also decreased suddenly from a peak. This occurred after 30% of the material had been removed and before it had reached 35%.

When all of the points denoted A, including point 8a (i.e., all of the points of 100% stiffness weighting) are looked at, it is clear from Fig. 8 that as more material is removed, the $C \times V$ term decreases. This validates that the specific stiffness of the structure increases for optimization based purely on the stiffness criteria.

On the other hand, the $C \times V$ term increases for all of the points denoted by B, including point 8g, and excluding the points for the 35 and 40% of material removed cases. This signifies that the specific stiffness decreases for optimization based purely on the first mode of frequency.

Points 9a–9g in Fig. 8 represent the natural frequencies and $C \times V$ terms for the model with 30% of the material removed for the given variations in criteria weighting. The corresponding design topologies can be seen in Fig. 9. Although it is true for all of the cases in Fig. 8 that any improvement of one criterion requires a clear tradeoff with the other, it is not so for this case. The topology based on a stiffness criterion weighting of 90% has a lower natural frequency than the topology based on a stiffness criteria weighting of 100% by 1.04%. Point 9a still has a lower $C \times V$ term than point 9b, as expected. It is, thus, optimum in terms of being the stiffest possible design.

The frequency and $C \times V$ value for the global criterion method of multicriterion ESO case is also presented in Fig. 8 (point 10), with the topology presented in Fig. 10. Here, as can be seen, this topology forms part of the Pareto solution for the case of 30% of the material removed. Again, it lies very near the point formed by the $w_{\text{stiff}} : w_{\text{freq}} = 0.5 : 0.5$ case from the weighting method.

VI. Conclusions

A new algorithm incorporating multiple criterion optimization into ESO using the weighting and global criterion method has been developed. The topologies produced by the weighting method form the Pareto solution space in the two models that have been analyzed. Those designs based purely on stiffness are optimally stiff for their given volume fraction and likewise with frequency. Those designs based purely on the first mode of natural frequency are optimal in frequency for their given volume fraction. Any improvement in one criterion requires a clear tradeoff with the other criteria.

The solutions of the global criterion method also form part of the Pareto solution space. For this method, there appears to be a definable position on the Pareto curve that can be determined for each model. This position lies very near the 50% stiffness: 50% frequency point or equally spaced between the two extremes of the criteria on the Pareto curve.

Although only two objective optimality constraints have been considered in this work, the procedure can be applied to other criteria including buckling and inertial properties. This is being investigated and shall be reported in the near future.

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